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Generalized Love waves

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Abstract. – We present a generalized dispersion equation for shear horizontally polarized acoustic waves in a system of a finite-thickness substrate covered by a finite-thickness solid layer having a lower shear acoustic speed. The solutions to the equation describe both layer-guided Love waves and resonant acoustic plate modes. We identify higher-order mode Love waves as continuations from the lowest-order acoustic plate mode associated with the previous Love wave mode. Experimental results are presented that confirm the relationship between the multiple Love wave modes and the acoustic plate modes.

In the more than thirty years since it was suggested that surface acoustic waves (SAWs) could be excited by an interdigital transducer [1], their utility as a tool to investigate fundamental properties of materials has been established. The Rayleigh SAW, with a mechanical component of displacement perpendicular to the surface, has been used to investigate transport properties of one- and two-dimensional electron gas systems, the fractional quantum-Hall effect and acoustoelectric charge transport in semiconductor quantum wells [2–5]. Shear horizontal SAWs, with their mechanical displacement in the plane of the surface, have also been studied in supported thin films on GaAs wafers and in adsorbed Si-SiO_2 bilayers [6,7]. In recent years, the use of SAWs has been significantly extended into chemical and biochemical applications. A fundamental requirement in these applications is for highly surface mass-sensitive techniques capable of operating in an aqueous environment. To avoid the high damping that may be caused by the aqueous environment the acoustic waves must be either shear horizontally polarized or have a phase speed less than the speed of sound in the liquid. A wide range of surface acoustic-wave types has been considered, including Love waves and shear horizontally polarized acoustic plate modes (SH-APM) [8]. Love waves were first reported for use as a biosensor in 1992 by Gizeli *et al.* [9] and offer one of the highest potential mass sensitivities, but they have generally been regarded as distinct from acoustic plate modes. Therefore, a physical understanding of the spectrum of shear horizontally polarized acoustic modes occurring in a Love wave configuration is of wide interest.

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A Love wave is a shear horizontally polarized acoustic wave that is localized to the surface of a semi-infinite half-space and guided by a layer which has a shear acoustic speed less than that of the half-space material [10, 11]. The phase velocity of the Love wave is intermediate between that of the substrate and the layer and is determined by the layer thickness. An acoustic plate mode occurs when a finite-thickness substrate is excited at a natural resonant frequency of the substrate [12, 13]. The lowest-order, $n = 0$, APM mode corresponds to a plane wave whilst higher-order APM modes correspond to resonances of the plate. In a Love wave the energy is localized near the surface of the substrate and the phase speed is less than that of the shear acoustic speed of the substrate. In contrast, in an APM the energy is distributed throughout the substrate and the phase speed is higher than that of the shear acoustic speed of the substrate.

In this letter, we generalize the theory of Love waves to a finite-thickness substrate and show that Love waves and acoustic plate modes are two branches of solutions of the dispersion equation. It is shown that each Love wave mode is matched by a set of acoustic plate modes. We also demonstrate that as the guiding-layer thickness increases, the phase speed of the $m = 1$ acoustic plate mode associated with a Love wave mode reduces until it transforms into the next higher-order Love wave mode. We also report new experimental results obtained for 312 MHz acoustic waves on a polymer-coated quartz device using an interdigital transducer with a high-frequency resolution. The spectrum of shear horizontally polarized modes close to the resonant frequency has been investigated for a wide range of polymer guiding-layer thickness. The observed changes in phase speed of the various acoustic modes in the spectrum support the theoretically predicted relationship between Love waves and SH-APMs.

We consider shear horizontal waves with displacements in the x_2-direction travelling along the x_1-direction in a system comprised of an isotropic non-piezoelectric material of thickness w, covered by an isotropic non-piezoelectric material of thickness d. A solution for the wave equation is sought by using displacements in the layer, $\underline{u}_{\mathrm{l}}$, and the substrate, $\underline{u}_{\mathrm{s}}$, of

$$\underline{u}_{\mathrm{l}} = (0,1,0)\lfloor Ae^{-jT_{\mathrm{l}}x_3} + Be^{jT_{\mathrm{l}}x_3}\rfloor e^{j(\omega t - k_1 x_1)}$$

and

$$\underline{u}_{\mathrm{s}} = (0,1,0)\lfloor Ce^{T_{\mathrm{s}}x_3} + De^{-T_{\mathrm{s}}x_3}\rfloor e^{j(\omega t - k_1 x_1)},$$

where ω is the angular frequency. The wave vector $k_1 = (\omega/v)^{1/2}$, where v is the phase speed of the Love wave. From the equations of motion, the wave vectors T_{l} and T_{s} are given by

$$T_{\mathrm{l}}^2 = \omega^2\left(\frac{1}{v_{\mathrm{l}}^2} - \frac{1}{v^2}\right) \qquad \text{and} \qquad T_{\mathrm{s}}^2 = \omega^2\left(\frac{1}{v^2} - \frac{1}{v_{\mathrm{s}}^2}\right),$$

where the substrate and layer shear speeds are related to the rigidities, μ_i, and densities, ρ_i, of the substrate and layer materials by $v_{\mathrm{s}} = (\mu_{\mathrm{s}}/\rho_{\mathrm{s}})^{1/2}$ and $v_{\mathrm{l}} = (\mu_{\mathrm{l}}/\rho_{\mathrm{l}})^{1/2}$. Requiring the solution to satisfy boundary conditions of continuity of displacement and stress at the various interfaces gives the dispersion equation [14]

$$\tan(T_{\mathrm{l}}d) = \xi \tanh(T_{\mathrm{s}}w),$$

where $\xi = \mu_{\mathrm{s}}T_{\mathrm{s}}/\mu_{\mathrm{l}}T_{\mathrm{l}}$.

The dispersion equation has two types of solution. The first has T_{s} real and non-zero and gives a series of modes with phase speeds lower than the shear acoustic speed of the substrate; these correspond to Love waves. The second has $T_{\mathrm{s}} = jk_{\mathrm{s}}$ with k_{s} real and gives a series of modes with phase speeds higher than the shear acoustic speed of the substrate; these correspond to acoustic plate modes. The thickness of the substrate, w, determines the

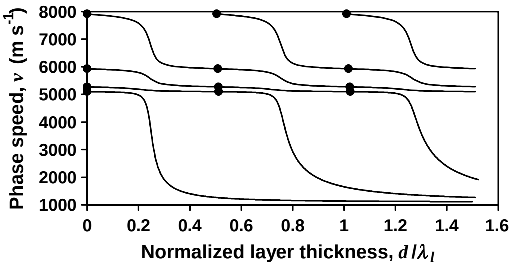

Fig. 1 – Theoretical calculated phase speeds showing multiple modes of Love waves and the associated acoustic plate modes. The solid circles indicate the analytical result for the start of each mode.

number of Love wave modes and the spacing of the associated acoustic plate modes. At the start of each successive Love wave mode the phase speed of the Love wave is $v = v_\mathrm{s}$ and the speeds of the associated plate modes are $v_\mathrm{m} = v_\mathrm{s}/(1 - (m\pi v_\mathrm{s}/w\omega)^2)^{1/2}$. The guiding-layer thickness can be chosen to simultaneously satisfy both the Love wave condition $\tan(T_\mathrm{l}d) = 0$ and the SH-APM condition $\tan(k_\mathrm{s}w) = 0$ by choosing a thickness d_{nm} given by

$$\frac{d_{nm}}{\lambda_\mathrm{l}} = \frac{n}{2\sqrt{1 - \left(\frac{v_\mathrm{l}}{v_\mathrm{s}}\right)^2 \left[1 - \left(\frac{m\lambda_\mathrm{s}}{2w}\right)^2\right]}},$$

where $n = 0, 1, 2, 3, \ldots$ labels the successive Love wave modes and $m = 1, 2, 3, \ldots$ labels the acoustic plate modes associated with each Love wave mode. For consistency with the start of a Love wave mode on a semi-infinite substrate, we can regard the thickness d_{nm} and the phase speed v_m as the start of a new mode. Traditional Love waves correspond to $m = 0$ whilst acoustic plate modes correspond to $m > 0$. As the guiding-layer thickness increases, the $n = 0$ Love wave evolves into a displacement with a single node located close to the substrate-layer interface and an antinode at the surface of the guiding layer. Each higher-order Love wave introduces an additional node within the layered system. Increasing the thickness of the guiding layer by an amount insufficient to change modes causes a reduction in the phase speed of the mode. Although it is not possible to analytically solve the dispersion equation, we have obtained the fractional change in phase speed caused by increasing the thickness of the guiding layer by an amount, Δd, above the guiding-layer thickness resulting in the start of a mode. In the case of a Love wave $\Delta v/v_\mathrm{s} \propto \omega^2 \Delta d^2$, assuming the substrate is sufficiently thick compared to the perturbation, whereas for the lower-order associated acoustic plate modes we find $\Delta v/v_\mathrm{s} \propto \Delta d/w$.

In fig. 1 we show the numerical results for the phase speed obtained from the dispersion equation using parameters of $w = 100\,\mu\mathrm{m}$, $f = 100\,\mathrm{MHz}$, $v_\mathrm{l} = 1100\,\mathrm{ms}^{-1}$, $v_\mathrm{s} = 5100\,\mathrm{ms}^{-1}$, $\rho_\mathrm{l} = 1000\,\mathrm{kgm}^{-3}$ and $\rho_\mathrm{s} = 2655\,\mathrm{kgm}^{-3}$. With the exception of the substrate thickness, these parameters describe high-frequency Love waves using a poly(methylmethacrylate) guiding layer on quartz. The substrate thickness in the calculation is thinner than typical substrates used in experiments, but this is solely to enable the acoustic plate modes to be resolved in fig. 1. The points shown in fig. 1 are the start of each mode calculated using the analytical results. Numerically, it is clear that each higher-order Love wave mode, labelled by $n = N$ and $m = 0$, arises as a continuation of the $m = 1$ acoustic plate mode, labelled by $n = N - 1$ and $m = 1$, associated with the previous Love wave mode. An alternative view is that the spectrum is a continuous evolution with v_m and d_{nm}, given by the points in fig. 1, defining the

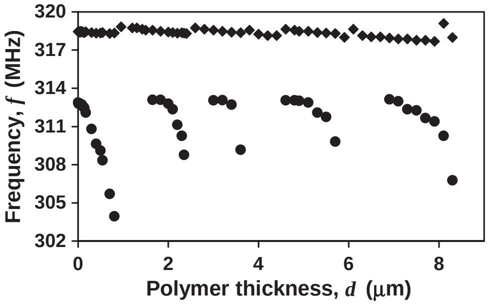

Fig. 2 – Experimental data for the frequency shift of shear horizontally polarized acoustic modes on a device operating at 312 MHz. The Love modes are indicated with the solid circles and the acoustic plate modes with the solid diamonds.

layer thickness at which an antinode in the displacement crosses from the substrate into the layer. Indeed, an alternative labelling of the modes would be to use a single index representing the total number of nodes in the complete substrate-layer system.

The experimental observation of the Love waves and associated acoustic plate modes used propagation orthogonal to the x-direction of an ST-cut quartz substrate which supports shear horizontally polarized acoustic waves [15]. The acoustic waves were generated using two sets of interdigital transducers separated by 7 mm to create a delay line device. The transducers were located on the polished side of the 0.5 mm thick quartz substrate. Each transducer had 120 pairs of double fingers with an electrical period of 16 μm and an aperture of 180.5 wavelengths. This arrangement gives a resonant frequency of 312 MHz with a bandwidth, $\Delta f/f_0$, of better than 1% for the transducer. The expected frequency separation between the m-th acoustic plate mode and the Love wave of frequency f_0 is given by $\Delta f/f_0 = (n\lambda/2w)^2/2$, where λ is the electrical period of the transducer. The layered system was created by successively spin-coating a polymer (S1813 photoresist from Shipley) at 4000 rpm across the whole device and then softbaking the device at 120 °C for 90 s on a hotplate. After each step the frequency spectrum of the device was measured (Agilent 8712ET network analyzer) and the resonant frequency of each mode recorded.

The observed change in frequency with polymer guiding-layer thickness is shown in fig. 2. Even a thin guiding layer causes a much larger downward shift in the frequency of the Love

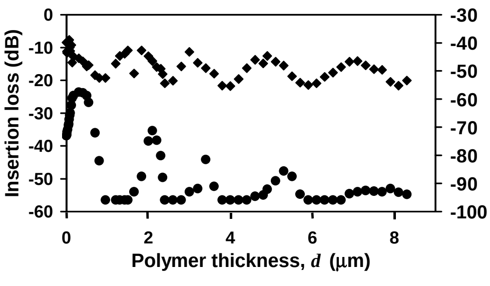

Fig. 3 – Experimental data for the insertion loss of shear horizontally polarized acoustic modes on a device operating at 312 MHz. The data for the Love waves is shown in circles and corresponds to the left-hand axis. The data for the acoustic plate modes is shown as diamonds and corresponds to the right-hand axis.

wave (circles) compared to the acoustic plate mode (diamond). The guiding layer also initially enhances the transmission of the Love wave, but as the layer thickness increases the mode is increasingly attenuated by the polymer layer until it can no longer be separated from the baseline noise (circle symbols and left-hand y-axis in fig. 3). The plate mode is also periodically attenuated as the polymer layer thickness increases with the attenuation reaching the baseline prior to the start of each new plate mode (diamond symbols and right-hand y-axis in fig. 3). Further increasing the polymer layer thickness eventually results in the appearance of the next Love wave mode with amplitude similar to that of the uncoated substrate. We have observed five Love wave modes with this type of device.

In conclusion, by developing a generalized dispersion equation for Love waves on a finite-thickness substrate we have determined a spectrum that also includes shear horizontally polarized acoustic plate modes. We have demonstrated that a higher-order mode of Love wave can be considered to arise from previously existing acoustic plate modes associated with the previous Love wave mode. We have shown experimentally, on a substrate without specific preparation of the back surface, that both multiple Love wave modes and the associated shear horizontally polarized acoustic plate modes can be excited.

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